Financial and Operations Decision Making of Risk-Averse Financial Institution and Risk-Neutral Retailer

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Abstract: Asset-based lending solidifies its position as a mainstream lending option for business. The issue of its impact on the operations decision has not received sufficient attention from academic community. This paper investigates the risk-neutral retailer's inventory decision without any loan limitation in a single period newsvendor model. The risk-averse lender's decision about the line of credit is derived. The research shows that less capital the retailer has more impact on the line of credit the degree of averseness has. The inventory value and the process of developing the advance rate can prevent retailers with less capital from obtaining the loan.

Keywords: Inventory financing; Leader-follower game; Line of credit; Conditional value-at-risk

I. Introduction

Corporate financing decisions have not obtained sufficient attention in the operations management literature which assumes that a firm's optimal inventory level or production decisions can be fully financed by the internal capital. In reality, firms, especially small- and middle-size firms, often face financial constraints, because their development heavily depends on the external capital. Conversely, the firm's operations decisions affect its borrowing capability. For example, in asset-based financing, the maximum amount of the loan is linked to the appraised value of the firm's assets in the form of inventory and accounts receivable.

According to a statistics compiled by the Commercial Finance Association, the U.S. asset-based lending (ABL) industry grew 16.5% in 2006 and approached the $500 billion level in terms of total asset-based loans outstanding at year-end. The asset-based lending industry has sustained steady growth over the past 5 years. The principal factors that determine the line of credit include the type of business and the content and quality of the collateral. In this paper, retailers regard the inventory as the collateral. The line of credit is determined according to the inventory cost at funding, the appraised orderly liquidation value (OLV), the orderly liquidation fixed cost, the orderly liquidation variable cost and loan advance rate (the ratio of the amount of loan to the net OLV).

This research investigates the risk-neutral retailer's inventory decision without loan limit in a single period newsvendor model. In the financial literature, it has been pointed out that to maximize the expected profit is not satisfactory from the practical point of view, and managers in the real world concern more on the risk measurements of risk. The risk measurement plays a crucial role in optimization under uncertainty, especially in coping with the losses that might be incurred in the finance or insurance industry. To the risk-averse lender, a mean-CVaR objective is built. We show that the higher the degree of adverseness is, the lower the line of credit is.

To our best knowledge, the issue of the impact of asset-based lending on the operations decision has not been discussed sufficiently. The exceptions are [1] and [2] which focus on the relationship between a risk-neutral retailer and a risk-neutral bank in a single period newsvendor model. While in our paper, we employ mean-CVaR objective to formulate the risk-averse lender's decision.

II. Risk-neutral Retailer's Decision

This research begins at the request of a seasonal commodity retailer, or a newsvendor. The retailer has initial cash of x_0 but no other asset. The retailer can place an order of size q_0 at a cost of c per unit from her suppliers. The size q_0 is no more than x_0/c with the budget constraint. In order to meet the future random demand d, the retailer appeals to the inventory lender for the purchase cost of the other order of size q. The lender funds the retailer with the inventory of size q_0+q as collateral. The collateral is in the charge of the lender. According to the finance literature on ABL (see [4]), five factors affect the line of credit: the inventory cost at funding $c(q+q_0)$, the appraised OLV$\alpha c(q+q_0)$, the orderly liquidation costs F_1, the orderly liquidation variable costs $\beta\alpha c(q+q_0)$ and loan advance rate γ. Therefore, the line of credit is determined to be $\gamma(\alpha c(q+q_0)- F_1-\beta\alpha c(q+q_0))$ (see Table I).

Table I. Sequence of Developing a Proper Advance Rate

Inventory cost at funding	$c(q+q_0)$
Appraised OLV at α of cost	$\alpha c(q+q_0)$
Orderly liquidation fixed costs including: Utilities, mortgage and Taxes	F_1
Orderly liquidation variable costs including: Advertising, site preparations, accounting, travel,	$\beta\alpha c(q+q_0)$

telephone, custodial, security, etc.	
Total orderly Liquidation expenses	$F_1+\beta\alpha c(q+q_0)$
Net OLV forecasted	$\alpha c(q+q_0)- F_1-\beta\alpha c(q+q_0)$
Loan advance rate γ of net OLV	$\gamma(\alpha c(q+q_0)- F_1-\beta\alpha c(q+q_0))$

When the demand comes, the retailer sells the commodity at a fixed per unit price p. The unsold inventory is disposed at a price of $c^{'}$, $c^{'}<c$, per unit. With a lockbox system, the receipts of customers are channeled directly to the lending institution instead of the retailer. At maturity, the retailer has to pay the lender $cq(1+r)$, the required loan repayment plus the required loan interest. If the retailer realizes that the revenue from the sales of the inventory cannot afford the loan, interest, the operational fixed cost F_0 and the operational variable cost $s(q_0+q)$, it announces bankruptcy. At the time, the lender can only dispose of the unsold inventory with the liquidation fixed and variable cost. If the revenue in the lockbox is more than the loan and interest, the lender returns the surplus revenue to the retailer, as well as the unsold inventory.

Assume that the lender and the retailer have the common belief about the probability distribution of demand d given by $F(u)$. Let $\overline{F}(u)=1-F(u)$ and $f(u)=F^{'}(u)$. The retailer is risk-neutral and can receive a credit of $r_0(x_0-cq_0)$ on the due date. Let r_0 be the deposit rate. Assume that $r_0<r$. For a given interest rate r, no retailer borrows if $p\le c(1+r)+s$. So we only consider the case where $p>c(1+r)+s$. The retailer's final cash position $x_T(d)$ with the demand of d after repaying the loan is given by

$$x_T(d)=(x_0-cq_0)(1+r_0)+p\min\{q_0+q,d\}+c^{'}\max\{q_0+q-d,0\}-cq(1+r)-s(q_0+q)-F_0$$

and the retailer is bankrupt if $x_T(d)<0$.

LEMMA 1. *It is optimal for the retailer to ensure that $(x_0-cq_0)q=0$ in the case where $r_0<r$.*

From Lemma 1, the retailer should use up all cash before approaching the lender. Due to our focus on the interaction of financial and operational decisions, we consider only the case with $q_0=x_0/c$ in the following.

According to Zhang et al. (2008), we can obtain the following expected cash position to the retailer.

$$E\left[x_T(\xi)\right]=\begin{cases} p(q_0+q)-cq(1+r)-s(q_0+q)-F_0 \\ -\int_0^{q_0+q}(p-c^{'})F(x)dx \quad if\ q\le\xi(q_0) \\ p(q_0+q)-cq(1+r)-s(q_0+q)-F_0 \\ -\int_{d(q_0,q)}^{q_0+q}(p-c^{'})F(x)dx \quad if\ q>\xi(q_0)\end{cases} \tag{1}$$

Where

$$\xi(q_0)=\max\left\{0,\ \frac{\left(c^{'}-s-f_0(q_0)\right)}{\left(s+c(1+r)-c^{'}\right)}q_0\right\}\ ;q_0=\frac{x_0}{c}$$

$$d(q_0,q)=\frac{F_0+(s-c')q_0+\left(s+c(1+r)-c'\right)q}{p-c^{'}}\ ;f_0(q_0)=\frac{F_0}{q_0}.$$

The following theorem identifies the order quantity a retailer chooses. The proof is in [5].

THEOREM 1. *For increasing failure rate (IFR) distributions of demand, the order quantity a retailer with initial capital of x_0 chooses, q^{R*}, for given r is as follows:*

1. If $\xi(q_0)>0$, then

$$q^{R*}=\begin{cases}\frac{x_0}{c}+q^{NB}-\frac{x_0}{c} & if\ cq^{BWO}\le x_0\le cq^{NB} \\ \frac{x_0}{c}+q\left(\frac{x_0}{c}\right) & if\ x_0\le cq^{BWO}\end{cases}$$

2. If $\xi(q_0)=0$, then

$$q^{R*}=\frac{x_0}{c}+q\left(\frac{x_0}{c}\right)\quad if\ x_0\le cq^{BWO32}$$

Where

$$F(q^{NB})=\frac{p-c(1+r)-s}{p-c^{'}};$$

$$q^{BWO}=\frac{\left(c(1+r)+s-c^{'}\right)}{c(1+r)}\left(q^{NB}+\frac{F_0}{s+c(1+r)-c^{'}}\right);$$

$$q^{BWO32}=\inf\{q_0:q(q_0)=0\}$$

$$\overline{F}\left(\frac{x_0}{c}+q\left(\frac{x_0}{c}\right)\right)=\frac{c(1+r)-c^{'}+s}{p-c^{'}}\overline{F}\left(d\left(\frac{x_0}{c},q\left(\frac{x_0}{c}\right)\right)\right)$$

Furthermore, $\frac{x_0}{c}+q(\frac{x_0}{c})$ *and* $q(\frac{x_0}{c})$ *are decreasing in x_0.* $q(q^{BWO})=q^{NB}-q^{BWO}$, $q^{BWO}<q^{NB}<q^{BWO32}$.

III. Risk-averse Lender's Decisions

The conditional value-at-risk (CVaR) is known as a risk measure which is coherent and consistent with the second (or higher) order stochastic dominance. In particular, the consistency with the stochastic dominance implies that minimizing the CVaR never conflicts with maximizing the expectation of any risk-averse utility function (see [3]).

We adopt the net loss defined by $cq(r-r_0)-\Pi(d)$ as the loss function L so that the lender can consider the loss lower than ω_δ. Therefore, the mean-risk model using the net loss CVaR is formulated as

$$\text{maximize}_{q,\omega}E\left[\Pi(d)\right]-\lambda\omega+\lambda\left(\frac{1}{1-\delta}\int_0^\infty\left[cq(r-r_0)-\Pi(x)-\omega\right]^+f(x)dx\right) \tag{2}$$

For $0\le q\le\xi(q_0)$, there is no bankruptcy risk, therefore objective (2) becomes

$$\text{maximize}_{q,\omega}E\left[\Pi(d)\right]-\lambda\omega \tag{3}$$

Where

$$E\left[\Pi(d)\right]=\begin{cases} cq(r-r_0) & \text{if } 0<q<\xi(q_0) \\ cq(r-r_0) \\ -(p-c')\int_0^{d(q_0,q)}F(x)dx-\left(m+nq\right)F\left(d(q_0,q)\right) \end{cases}$$

and m=F_1-F_0+(βc-s)q_0, n=βc-s. Given the fact that the fixed and variable cost are no less than the running expenses of the retailer when the lender disposes of the inventory, we assume that $F_1 \geq F_0$ and $\beta c \geq s$. According to the equation of $E\left[\Pi(d)\right]$, we can obtain the following lemma.

LEMMA 1. *If* $0\le q\le \xi(q_0)$, $q^*=\xi(q_0)$, $\omega^*=0$ *; If* $q\geq\xi(q_0)$*, we have that*

1. $\Phi(0|q)=P\{L(q,d)\le 0\}=1-F(d(q_0,q))$ *;*
2. $\Phi(m+nq|q)=P\{L(q,d)<m+nq\}=1-F(d(q_0,q))$ *;*
3. $\Phi(cq(1+r)-c'(q_0+q)+F_1+\beta c(q_0+q)|q) = P\{L(q,d)\le cq(1+r)-c'(q_0+q)+F_1+\beta c(q_0+q)\} = 1$.

According to $\Phi(\eta|q)=P\{L(q,d)\le\eta\}$ and the equation of $\Pi(d)$, the Lemma follows.

Because $\Pi(d)=cq(r-r_0)$ for $d>d(q_0,q)$, we have that

$$\text{maximize}_{q,\omega} E\left[\Pi(d)\right]-\lambda\omega+\frac{\lambda}{1-\delta}\int_0^{d(q_0,q)}\left[cq(r-r_0)-\Pi(x)-\omega\right]^+ f(x)dx \tag{4}$$

LEMMA 2. *For* $\delta\in\left[0,1\right]$ *and* $q\in\left[\xi(q_0),+\infty\right]$*, the optimal* $\omega^*(q,\delta)$ *is given by*

1. *when* $q<q^{ba}(q_0)$, $\omega^*(q,\delta)=0$ *;*
2. *when* $q\ge q^{ba}(q_0)$,

$\omega^*(q,\delta)=cq(1+r)-c'(q_0+q)+\beta c(q_0+q)-(p-c')F^{-1}(1-\delta)$

where $F\left(d\left(q_0,\ q^{ba}(q_0)\right)\right)=1-\delta$.

To prove the lemma, we firstly consider three cases to estimate the integral in (4):

1. $\omega\ge cq(1+r)-c'(q_0+q)+F_1+\beta c(q_0+q)$;
2. $\omega\in\left[m+nq,\ cq(1+r)-c'(q_0+q)+F_1+\beta c(q_0+q)\right]$;
3. $\omega\le m+nq$.

Then based on the first-order condition, we can obtain Lemma 2. From Lemma 2, Problem (4) can be transformed into the following problem.

$$\max_q\begin{cases} E\left[\Pi(d)\right] & \text{if } \max\left\{\xi\left(q_0\right),0\right\}\le q<q^{ba}(q_0) \\ E\left[\Pi(d)\right]-\lambda\left(\omega^*(q,\delta)+\dfrac{p-c'}{1-\delta}\displaystyle\int_0^{F^{-1}(1-\delta)}F(x)dx\right) \end{cases} \tag{5}$$

THEOREM 2. *For increasing failure rate (IFR) distributions of demand, problem (5) with* $\delta\in\left[0,1\right]$ *has an optimal solution* (q^{ba*},ω^*) *defined as follows without loan limit.*

Case 1. $q^{ba}(q_0)>\max\{\xi(q_0),0\}$

1. *If* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=\max\{\xi(q_0),0\}}\le 0$*, then* $q^{ba*}=\max\left\{\xi\left(q_0\right),0\right\}$ *and* $\omega^*=0$ *;*
2. *If* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=\max\{\xi(q_0),0\}}>0$ *and* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=q^{ba}(q_0)}\le 0$*, then* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=q^{ba*}}=0$ *and* $\omega^*=0$ *;*
3. *If* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=q^{ba}(q_0)}>0$ *and* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=q^{ba}(q_0)}\le\lambda\left(cq(1+r)-c'+\beta c\right)$*, then* $q^{ba*}=q^{ba}(q_0)$ *and* $\omega^*=m+nq$ *;*
4. *If* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=q^{ba}(q_0)}-\lambda\left(cq(1+r)-c'+\beta c\right)>0$*, then*

$$\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=q^{ba*}}-\lambda\left(cq(1+r)-c'+\beta c\right)=0$$
$$\omega^*=cq(1+r)-c'(q_0+q)+F_1+\beta c(q_0+q)-(p-c')F^{-1}(1-\delta).$$

Case 2. $q^{ba}(q_0)\le\max\{\xi(q_0),0\}$

1. *If* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=\max\{\xi(q_0),0\}}\le 0$*, then* $\omega^*=0$ *and* $q^{ba*}=\max\left\{\xi\left(q_0\right),0\right\}$*;*
2. *If* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=\max\{\xi(q_0),0\}}>0$ *and* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=\max\{\xi(q_0),0\}}\le\lambda\left(cq(1+r)-c'+\beta c\right)$ *then* $q^{ba*}=\max\left\{\xi\left(q_0\right),0\right\}$ *and* $\omega^*=0$ *;*
3. *If* $\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=\max\{\xi(q_0),0\}}>\lambda\left(cq(1+r)-c'+\beta c\right)$ *,then*

$$\frac{\partial E\left[\Pi(d)\right]}{\partial q}\Big|_{q=q^{ba*}}-\lambda\left(cq(1+r)-c'+\beta c\right)=0$$
$$\omega^*=cq(1+r)-c'(q_0+q)+F_1+\beta c(q_0+q)-(p-c')F^{-1}(1-\delta).$$

From the first-order condition of Problem (5) and Theorem 2, the theorem follows.

PROPOSITION 1. 1. *The risk-averse lender's optimal quantity decreases as the averse degree increases. That is,* q^{ba*} *decreases as* λ *increases. 2. The risk-averse lender's optimal quantity decreases as confidence level increases. That is,* q^{ba*} *decreases as* δ *increases.*

THEOREM 3. *For increasing failure rate (IFR) distributions of demand, the optimal line of credit is* $t^*(q_0)cq_0$ *.* $t^*(q_0)$ *is given by*

1. *for retailers with* $q_0\le q^{BWO}$,

$$t^*(q_0)=\min\left\{\frac{q^{ba*}(q_0)}{q_0},\frac{q(q_0)}{q_0}\right\};$$

2. *for retailers with* $q^{BWO} \le q_0 \le q^{NB}$, $t^*(q_0) = \dfrac{(q^{NB} - q_0)}{q_0}$.

PROOF. According to $\dfrac{\partial E[\Pi(d)]}{\partial t} = \dfrac{\partial E[\Pi(d)]}{\partial q} q_0$ and Theorems 1 and 2, the result can be obtained. □

Denote $P(q_0, q, d) = p\min\{d, q_0 + q\} + c' \max\{q_0 + q - d, 0\}$.

$E[P(q_0, q, d)] = p(q_0 + q) - (p - c')\int_0^{q_0+q} F(x)dx$.

According to the following three different steps of calculating the net OLV, we analyze the impact of the parameters α, β and F_1 on the advance rate γ.

Case 1. α, β and F_1 are fixed. β and F_1 are set to equal the actual costs incurred at the liquidation time. The net OLV is equal to $\alpha c(q_0 + q) - F_1 - \alpha\beta c(q_0 + q)$.

Case 2. β and F_1 are fixed. β and F_1 are set to equal the actual costs incurred at the liquidation time. $\alpha = \dfrac{E[P(q_0, q, d)]}{c(q_0 + q)}$. The net OLV is equal to $E[P(q_0, q, d)]$ $-(F_1 + \beta E[P(q_0, q, d)])$.

Case 3. β and F_1 are fixed. β and F_1 are calculated when the retailer is bankrupt. That is, the net OLV is equal to $E[P(q_0, q, d)] - (F_1 + \beta c(q_0 + q))F(d(q_0, q))$.

Figure 1 shows the dependencies of the line of credit $t^*(q_0)cq_0$ and loan to value $t^*(q_0)cq_0/c(q_0+q)$ on x_0/c for p=120, c=60, c'=40, s=20, r=0.1, r_0=0.05, F_0=2000, β=0.4, F_1=2500, and normal random demand with mean μ=1000 and variance σ=300.

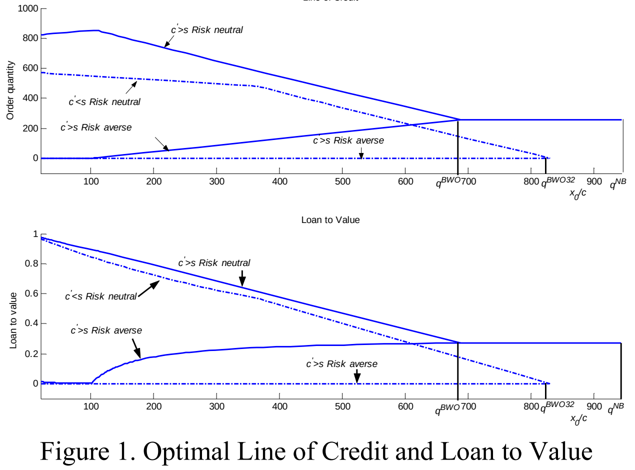

Figure 1. Optimal Line of Credit and Loan to Value

Figure 2 shows the dependencies of advance rate γ on x_0/c in the above three cases.

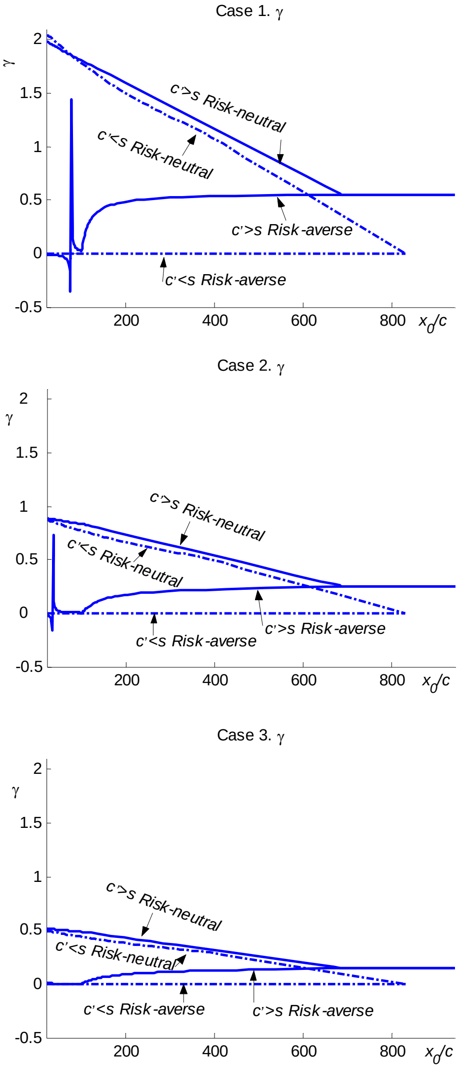

Figure 2. Advance Rate in Three Cases

Ⅳ. Conclusions

Considering that asset-based lending has solidified its position as a mainstream lending option for businesses, and the fact that the retailing industry represented 9.7% of total outstandings in 2006 and Retail-Department stores were among the major client sectors noted by individual lenders, and also that asset-based lending has not obtained sufficient attention in operations literature, we address the interaction between inventory management and the line of credit management.

We formulate the risk-neutral retailer's inventory decision model for maximizing its expected cash position without loan limit. According to the analysis of different cases, we derive the optimal solutions. Then we model the risk-averse lender's decisions about the line of credit without considering the decision of the retailer. To the risk-averse lender, the line of credit decreases as the averse degree and confidence level increase. Risk-averse lenders with 100% confidence level do not offer the loan to retailers in the commodity industry satisfying that the salvage price is lower than the variable cost. The inventory value and the process of developing the advance rate can prevent poorer retailers from obtaining the loan.

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